A Synopsis of Relative Expansion

By Jack Martinelli, August 9, 1998

In most theoretical treatments, Physicists typically do not consider a frame's intrinsic unit length in constructing their theories. Instead, quantities relating to length are expressed as meters or refer directly or indirectly to the speed of light (and time) as a reference length. Neither one is a direct representation of a frame's natural unit length. Simply defining c=1 is a step in the right direction but it is not clear how this relates to physical structure in a fundamental way.

It is plausible that a fundamental understanding of a frame's intrinsic or natural unit length could be useful. Defining this intrinsic length as an abstraction is easy… relating it to a meter -- a unit of material length – is not.

The following summarizes the development of length introduced in http://www.martinelli.org/fundamental to represent new expressions (isomorphs?) for length, velocity and acceleration as:

Length: sr

Velocity: sc

Acceleration: sa

The s represents the scale or linear density of a frame, r, represents measured length, c represents velocity and a is acceleration. In [ref] it is shown that:

$$s_0 r_0 = s_i r_i \qquad (1)$$

$$s_0 c_0 = s_i c_i \qquad (2)$$

$$s_0 a_0 = s_i a_i \qquad (3)$$

where the subscript denotes some frame number. s represents the scale of a frame. r represents a unit length, c an expansion/contraction velocity with respect to frame i and a the acceleration of the expansion.

We can use (1) and (2) to calculate an acceleration relationship expressed in (3).

Write (1) as: $s_1 = \frac{s_0 r_0}{r_1}$ and substitute it into (2) to get:

$$s_0 c_0 = \frac{s_0 r_0 c_1}{r_1} \qquad (4)$$

Which is constant.

Then take the derivative with respect to time to get:

$$a_1 = \frac{c_1^{\,2}}{r_1} \qquad (5)$$

In other-words, the representation for the acceleration of an expanding frame is exactly equivalent to the acceleration of uniform circular motion – i.e., the simplest possible oscillation that we know of. But it is not oscillating… there is only the abstract equivalence.

Now if we plug this back into (3) we have:

$s_0 a_0 = \frac{s_1 c_1^{\,2}}{r_1}$ And from (4) we have:

$$s_0 a_0 = \frac{s_0 r_0 c_1^{\,2}}{r_1^{\,2}} \qquad (6)$$

When we treat the expansion of a frame via special relativity, we account for its length contraction. (note that (6) is constant.)

$$l' = \frac{l c_0}{\sqrt{c_0^{\,2} + H^2 l^2}} \qquad (6.1)$$

defines the relationship of arbitrary lengths in the primed and unprimed frames under relative expansion.

Our expansion velocity for some expansion constant H then becomes:

$$c_1 = H \frac{l c_0}{\sqrt{c_0^{\,2} + H^2 l^2}} \qquad (7)$$

Note that as l approaches infinity that c_1 approaches c_0. Also note that for this relatively large l that the space of the frame is spherically curved with respect to the frame that it is contracting with respect to.
Then, using this result we can write (6) as:

$$sa = \frac{s_0 r_0 H^2 c_0^{\ 2}}{c_0^{\ 2} + H^2 r_1^{\ 2}} \qquad (8)$$

By inspection, you can see that this expression is not constant for all r_1 (as was the case in (6)), so we drop the subscripts from the *sa* term. If you plot this function you get:

Plot of sa over r shows a hump. (set r=0 &you get a maximum)

Note that the most significant feature of this field is that it is solitonic in form. I.e., a one hump wave.

Note that for $Hr_1 >> c_0$ that this becomes:

$$sa = \frac{s_0 r_0 c_0^{\ 2}}{r_1^{\ 2}} \qquad (9)$$

Which is the familiar form of the inverse square law (note the constant numerator and non-constant denominator).

Then integrating (9) over r we get the expression for work done by a contracting frame as:

$$E = -sc_0^{\ 2}$$

Then, since the acceleration of a contracting frame is abstractly equivalent to uniform circular motion (5), we can express radius $r_1 = c_0 / w$ … *as if* angular rotation were present. Then, substituting this into our field equation and substituting some $\hbar$ (…just a label for some constant) for $s_0 r_0 c_0$ we can calculate the second form of energy as: $E = -hf$, where $h = 2p\hbar$. This gives something that looks like that famous equation that Einstein found: $hf = sc_0^{\ 2}$ (Except that in Einstein's equation he used mass instead of scale.). That is, the "energy" of a contracting frame has two equivalent forms. One form is due to relativistic expansion, the other is due to a *mathematical equivalence* to angular rotation. Expanding *h* we can write: $2ps_0 r_0 c_0 f = sc_0^{\ 2}$ canceling c's and from (1) we have: $2ps_0 r_0 = sc/f$. This can also be written as: $s_0 l_0 = s_1 l_1$. In other words, the usual idea of wavelength is equivalent to a simpler idea: a frames's natural unit of length. See also the derivation of Snell's law in [ref]

So here's the essence of the problem I'm facing now. (6.1) describes how length is compressed. (9) expresses how "points" in a frame accelerate with respect to some origin. The question is: "how do you arbitrary represent motion of one frame with respect to another?"